Answers to Coursebook questions – Chapter G2

1 **a** The focal point of a converging lens is that point on the principal axis where a ray parallel to the principal axis refracts through, after passage through the lens.

b The focal length is the distance of the focal point from the middle of the lens.

2 **a** A real image is an image formed by actual rays of light which have refracted through a lens.

b A virtual image is formed not by actual rays but by the intersection of their mathematical extensions.

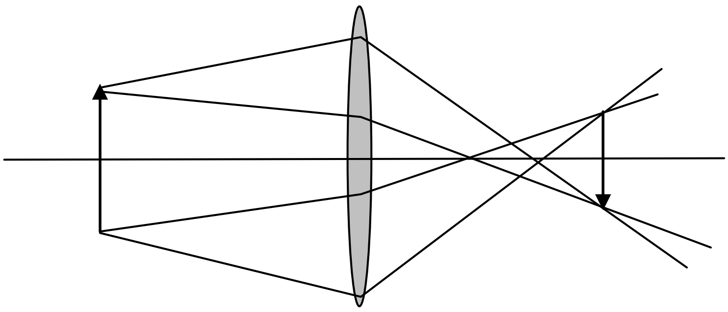
3 If a screen is placed at the position of a real image the actual rays of light that go through that image will be reflected off the screen and so the image will be seen on the screen. In the case of a virtual image, placing a screen at the position of the image reveals nothing as there are no rays of light to reflect off the screen.

4 No it does not – mirrors rely on reflection and lenses on refraction.

5 For the telescope to have a large magnification, a large objective lens focal length is required since $M = \frac{f_o}{f_e}$.

6 The incident rays are parallel so they will be brought to focus at a point on the focal plane. Hence the distance x is equal to the focal length of 6.0 cm.

7



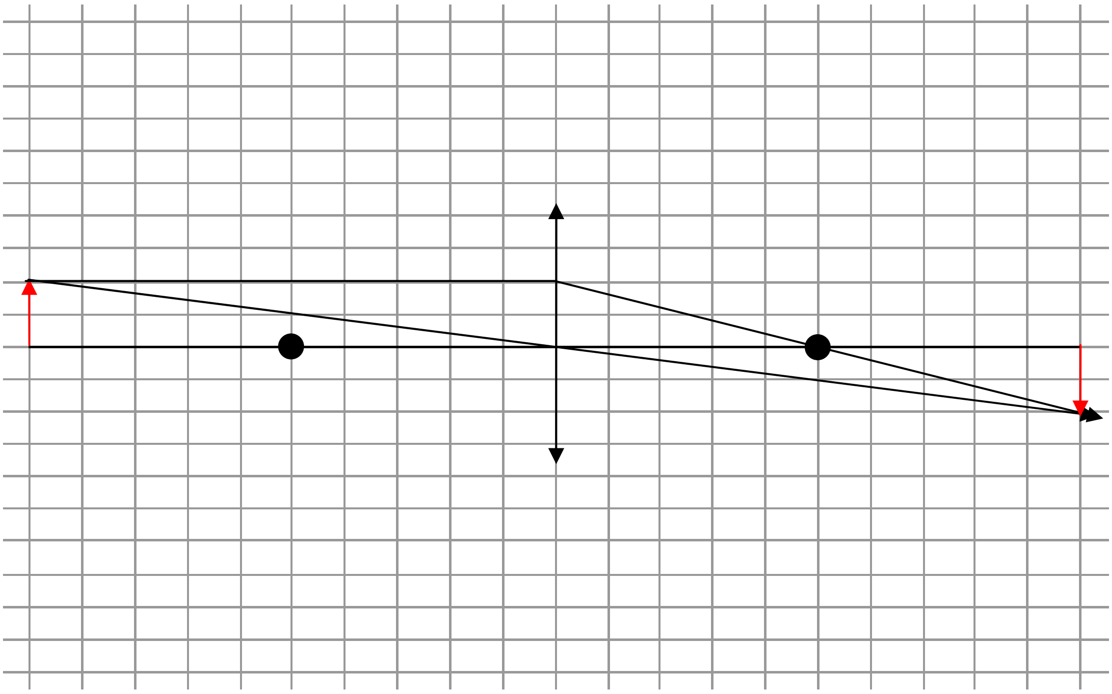
8 **a** The diagrams use a vertical scale of 1 cm per line and a horizontal scale of 2 cm per line.

$u = 20\text{ cm}$

The formula gives:

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \Rightarrow v = +20\text{ cm}$. Further, $M = -\frac{v}{u} = -\frac{20}{20} = -1$.

So the image is real (positive *v*), 20 cm on the other side of the lens, and the image is inverted (negative *M*) and has height 2 cm ($|M| = 1$). This is what the ray diagram below also gives.

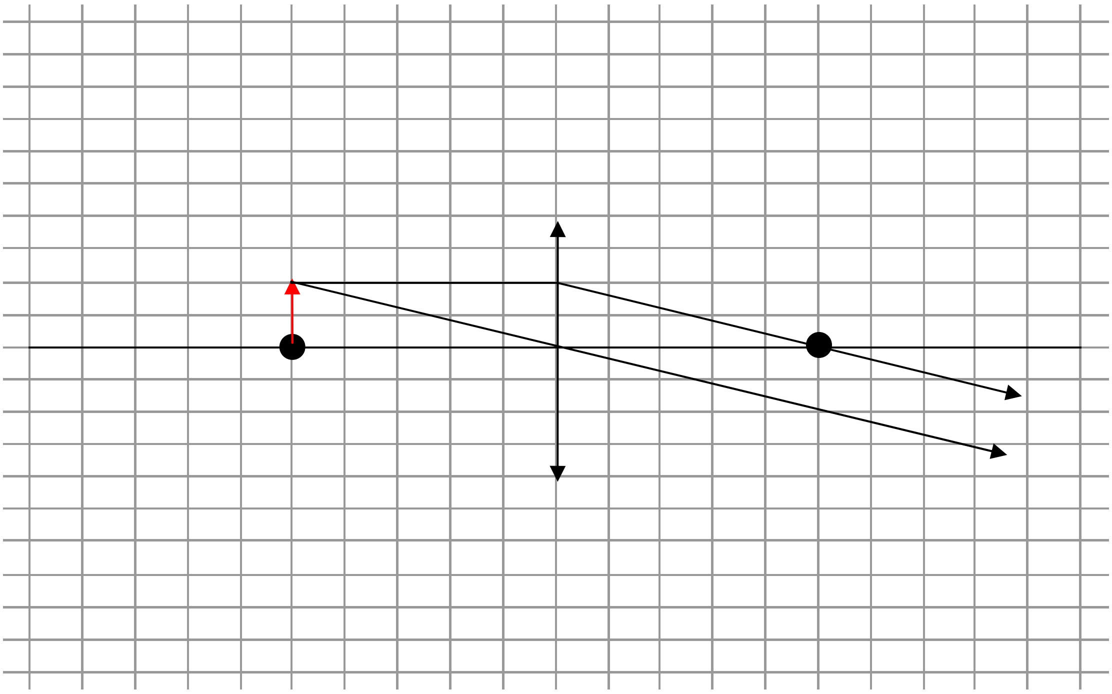


b $u = 10$ cm

The formula gives:

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{10}-\frac{1}{10}=0 \Rightarrow v=\infty .$$

The image is formed at infinity. This is what the ray diagram gives.



Rays do not meet even when they are extended. Image is said to ‘form at infinity’.

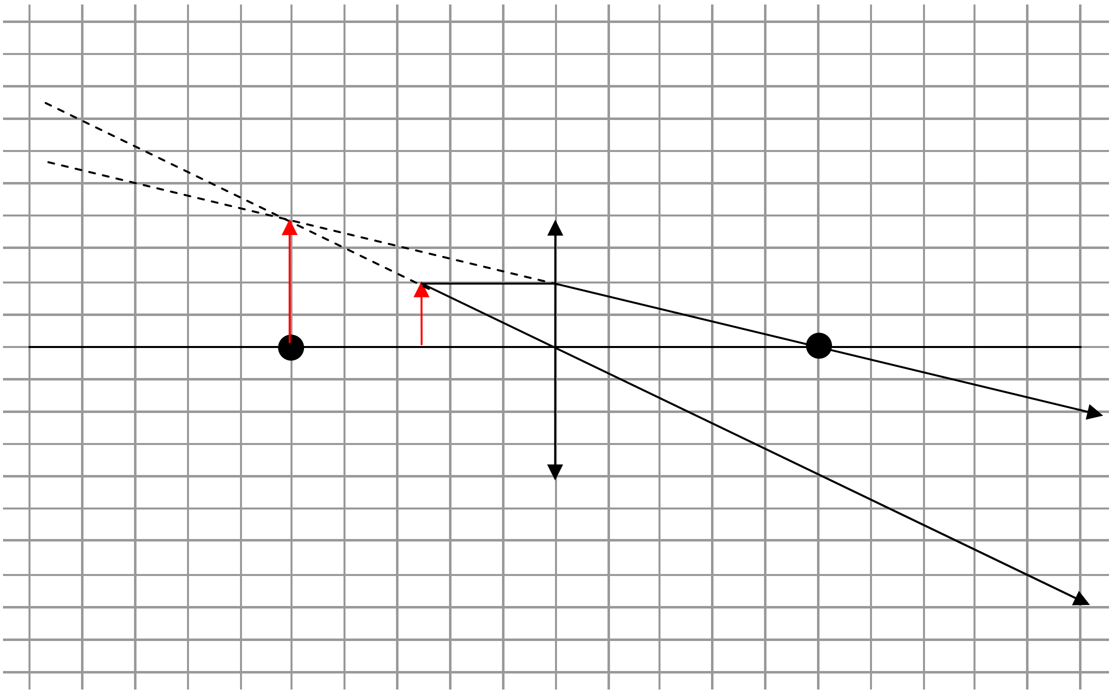
c $u = 5.0$ cm.

The formula gives:

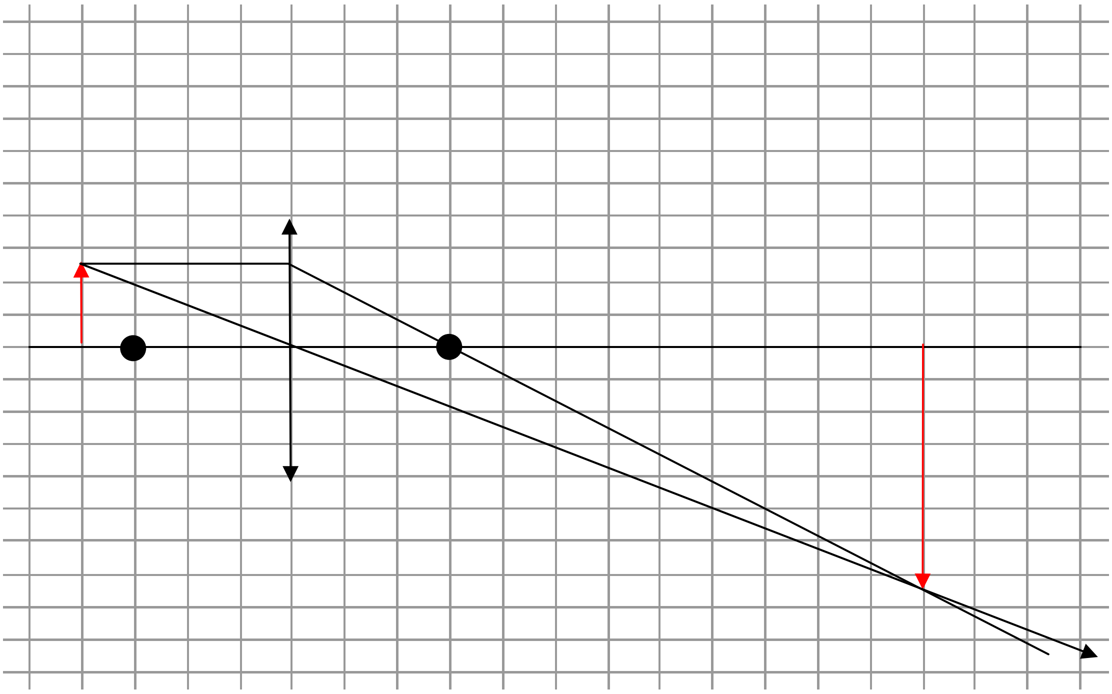
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{5.0} = -\frac{1}{10} \Rightarrow v = -10\text{ cm}.$$

Further $M = -\frac{v}{u} = -\frac{-10}{5.0} = +2$.

So the image is virtual (negative *v*), 10 cm on the same side of the lens, and the image is upright (positive *M*) and has height twice as large (i.e. 4 cm) (because $|M| = 2$). This is what the ray diagram below also gives.



9 The diagram uses a vertical scale of 1 cm per line and a horizontal scale of 2 cm per line.

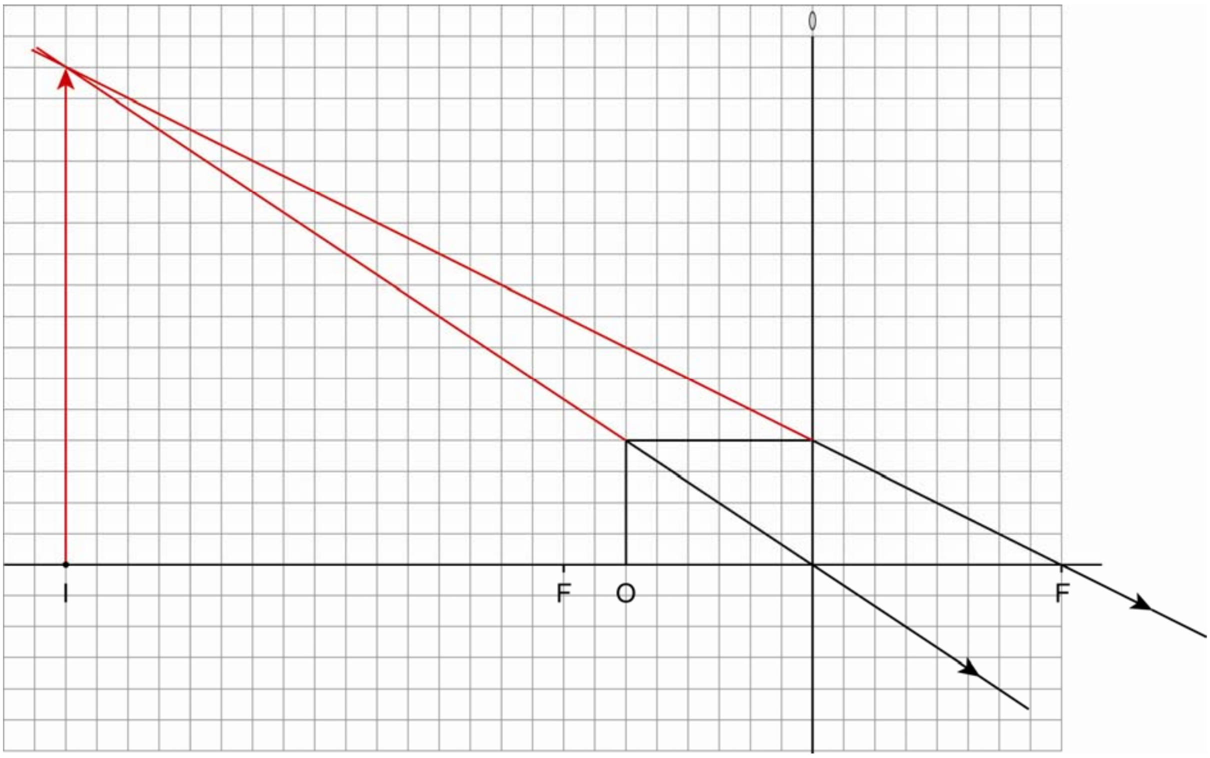


The formula gives:

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{6.0}-\frac{1}{8.0}\Rightarrow v=+24\text{ cm}\text{. Further, } M=-\frac{v}{u}=-\frac{24}{8.0}=-3\text{.}$$

So the image is real (negative *v*), 24 cm on the other side of the lens, and the image is inverted (negative *M*) and has height 3 as large (i.e. 7.5 cm) (because $|M|=3$). This is what the ray diagram above also gives.

10 The diagram uses a vertical scale of 1 cm per line and a horizontal scale of 1 cm per line.



The formula gives:

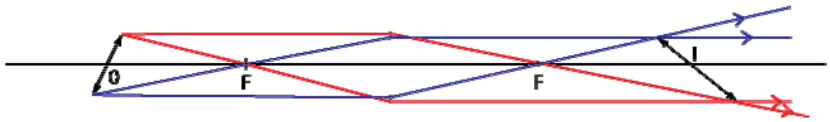
$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{8.0}-\frac{1}{6.0}=-\frac{1}{24}\Rightarrow v=-24\text{ cm}$$

Further

$$M=-\frac{v}{u}=-\frac{-24}{6.0}=+4$$

So the image is virtual (negative v), 24 cm on the same side of the lens, and the image is upright (positive M) and has height 4 as large (i.e. 16 cm) (because $|M|=4$). This is what the ray diagram below also gives.

11 See the diagram that follows. The rays from the top of the object have been drawn red and those from the bottom blue for the sake of clarity. The diagram shows the image tilts in the opposite direction to the vertical, compared to the object.



The top of the image will be formed at a distance form the lens given by:

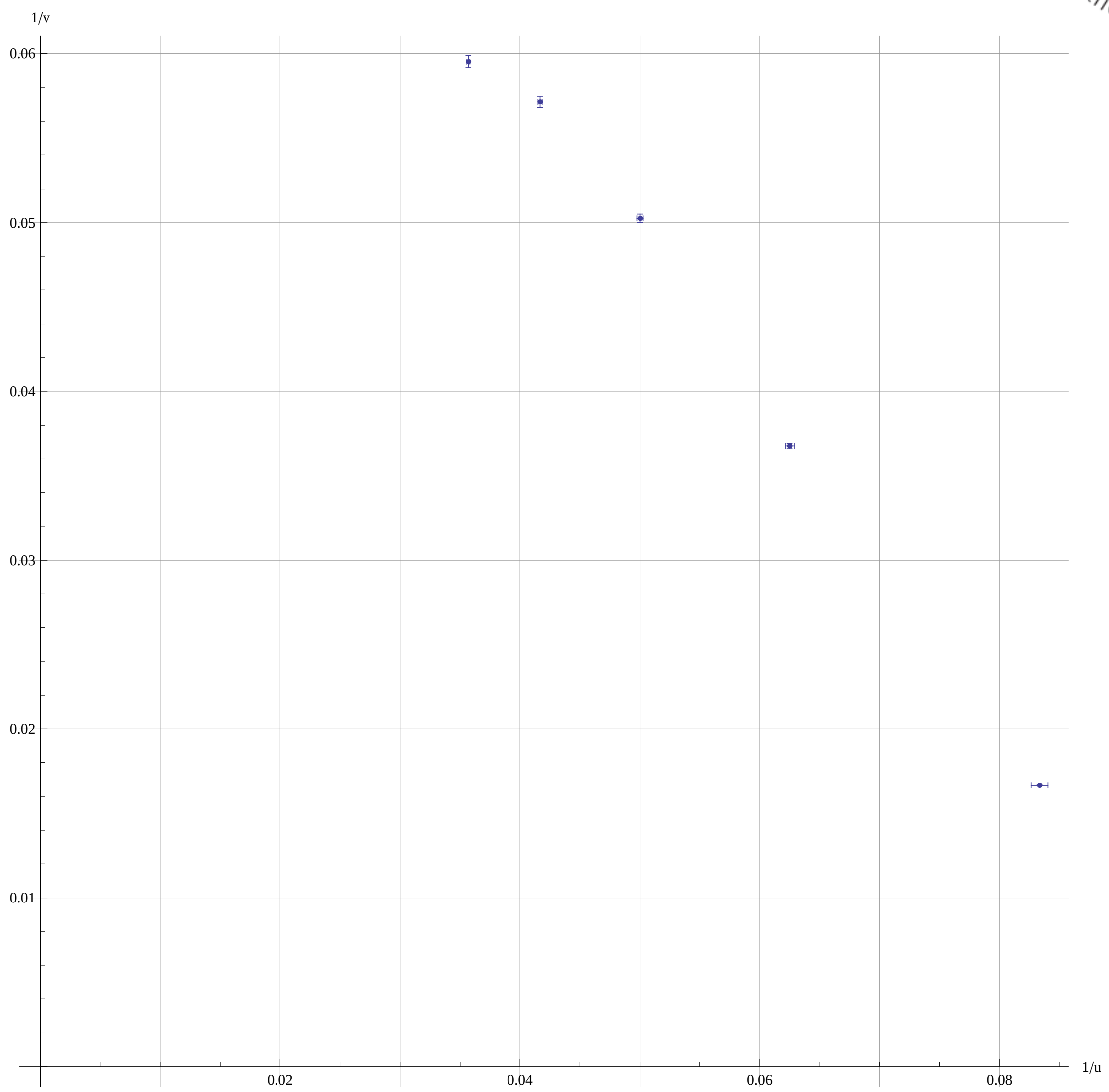
$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{5.0}-\frac{1}{9.0}\Rightarrow v=11.25\text{ cm}$ and the bottom at a distance of

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{5.0}-\frac{1}{10.0}\Rightarrow v=10.0\text{ cm}$.

To make the arithmetic a bit easier we will ignore that the length of the object is the one given and take instead what the diagram gives. Then the angle the object makes with the horizontal is $\tan^{-1}\frac{4}{1}\approx 76^{\circ}$. The image makes an angle given by $\tan^{-1}\frac{4.7}{1.25}\approx 75^{\circ}$ so it is slightly smaller.

12 **a** Since we know that $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ we should plot $\frac{1}{u}$ versus $\frac{1}{v}$. We expect a straight line with gradient equal to -1 and equal vertical and horizontal intercepts equal to $\frac{1}{f}$.

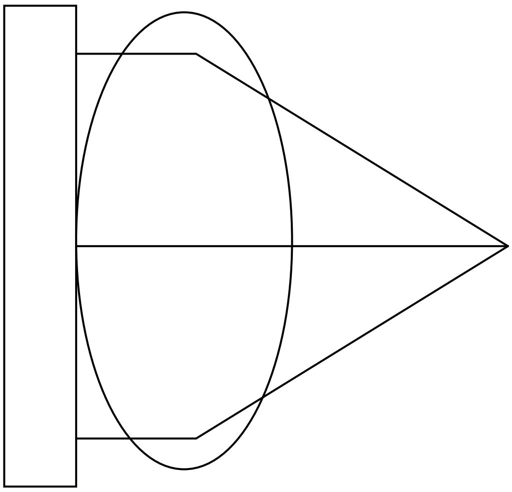
b The graph shows the data points and the (small) error bars.



The line of best fit is $\frac{1}{v} = 0.096 - \frac{0.95}{u}$.

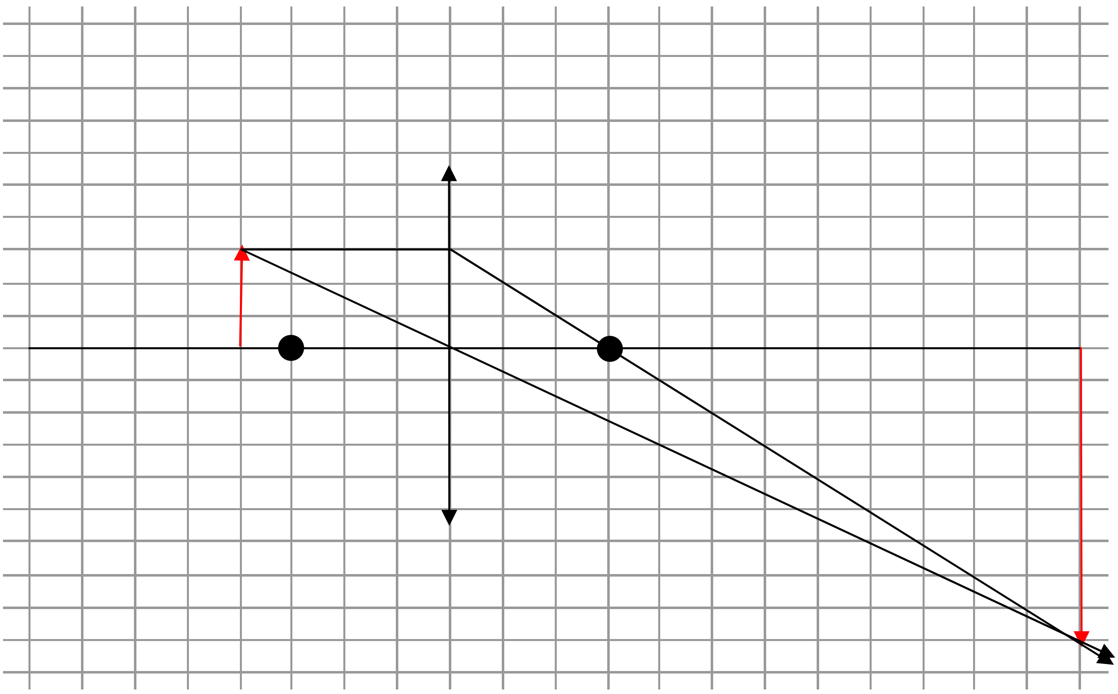
The intercepts are 0.096 and $\frac{0.096}{0.95}$ giving focal lengths $\frac{1}{f} = 0.096 \Rightarrow f = 10.42\text{ cm}$ and $\frac{1}{f} = \frac{0.096}{0.95} \Rightarrow f = 9.896\text{ cm}$. So approximately, $f = 10.2 \pm 0.3\text{ cm}$.

13 From the diagram it should be clear that rays must hit the mirror at right angles if they are to return to the position of the object. This means that the distance of the object from the lens when the object and image coincide is the focal length.



14 **a** $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \frac{1}{20} \Rightarrow v = +60\text{ cm}$. Further $M = -\frac{v}{u} = -\frac{60}{20} = -3$. So the image is real (negative *v*), 60 cm on the other side of the lens, and the image is inverted (negative *M*) and has height 3 as large (because $|M| = 3$). This is what the ray diagram below also gives.

b



15 **a** We must have that $u+v=5$ and $\frac{1}{u}+\frac{1}{v}=\frac{1}{0.6}$ where distances are in metres.

Then $v=50-u$ and so $\frac{1}{5-v}+\frac{1}{v}=\frac{1}{0.6}$. This gives $v^2-5v+3=0$ with solutions $v=4.30\text{ m}$ or $v=0.70\text{ m}$.

b In the first case the magnification is $M=-\frac{4.30}{0.70}=-6.1$ and in the second $M=-\frac{0.70}{4.30}=-0.16$ so the magnification is larger (in magnitude) in the first case.

16 To find the focal length we need to find where the lens combination would form the image of a very distant object, $u=\infty$. The first lens would form an image of this object at a distance from the first lens of $\frac{1}{\infty}+\frac{1}{v}=\frac{1}{10}\Rightarrow v=10\text{ cm}$, i.e. a distance from the second lens of 6.0 cm. But this object is to the right of the second lens so this is a virtual object, i.e. $u-6.0\text{ cm}$ for the second lens. The image is the found from $\frac{1}{u}+\frac{1}{v}=\frac{1}{10}\Rightarrow\frac{1}{-6.0}+\frac{1}{v}=\frac{1}{10}\Rightarrow v=3.75\text{ cm}$ to the right of the second lens. And this is where the focal point of the combination is.

17 **a** The image in the first lens is found from:

$\frac{1}{u}+\frac{1}{v}=\frac{1}{10}\Rightarrow\frac{1}{40.0}+\frac{1}{v}=\frac{1}{15.0}\Rightarrow v=24.0\text{ cm}$. This means that the distance of the image from the second lens is 1.0 cm. This image now serves as the object for the second lens. So $\frac{1}{1.0}+\frac{1}{v}=\frac{1}{2.0}\Rightarrow v=-2.0\text{ cm}$. The final image is 2.0 cm to the left of the second lens.

b The overall magnification is the product of the individual lens magnifications, i.e.

$$M=M_1M_2=\left(-\frac{24}{40}\right)\left(-\frac{-2.0}{1.0}\right)=-1.2.$$

c Since $M<0$, the final image is inverted relative to the original object and is 1.2 times larger.

18 **a** The image in the first lens is found from:

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}\Rightarrow\frac{1}{30.0}+\frac{1}{v}=\frac{1}{35.0}\Rightarrow v=-210\text{ cm}$. This means that the distance of the image from the second lens is 235 cm. This image now serves as the object for the second lens. So $\frac{1}{235}+\frac{1}{v}=-\frac{1}{20.0}\Rightarrow v=-18.4\text{ cm}$. The final image is 18.4 cm to the left of the second lens.

b The overall magnification is the product of the individual lens magnifications, i.e.

$$M=M_1M_2=\left(-\frac{-210}{30}\right)\left(-\frac{-18.4}{235}\right)=+0.548\approx 0.55.$$

c Since $M > 0$, the final image is upright relative to the original object and is 0.55 times smaller.

19 The light collected is proportional to the area of the collecting instrument and so the telescope can collect $\left(\frac{5.0}{3\times10^{-3}}\right)^2 = 2.78\times10^6 \approx 3\times10^6$ times more.

20 **a** The image is virtual so $v = -25\text{ cm}$.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{10.0}\Rightarrow\frac{1}{u}-\frac{1}{25}=\frac{1}{10.0}\Rightarrow u=7.143\approx7.1\text{ cm}.$$

b At the focal point of the lens, 10 cm away.

c The angular magnification in this case is $M=\frac{25}{f}=\frac{25}{10}=2.5$ and $M=\frac{\theta'}{\theta}$.

Now $\theta\approx\frac{1.6\times10^{-3}}{0.25}=0.0064\text{ rad}$ and so $\theta'\approx2.5\times0.0064=0.016\text{ rad}$.

21 **a** See **diagram G2.20** (page 617 in *Physics for the IB Diploma*).

b The nearest point to the eye where the eye can focus without straining.

c When the image is formed at the near point (25 cm away) we have that $v=-25\text{ cm}$.

Hence $\frac{1}{u}+\frac{1}{-25}=\frac{1}{f}\Rightarrow\frac{1}{u}=\frac{1}{f}+\frac{1}{25}=\frac{25+f}{25f}\Rightarrow u=\frac{25f}{25+f}$.

The magnification is then $M=-\frac{v}{u}=-\frac{-25}{\frac{25f}{25+f}}=+\frac{25+f}{f}=1+\frac{25}{f}$.

22 We are told that the smallest angle that can be resolved by the eye at the near point is $\theta\approx\frac{0.12\times10^{-3}}{0.25}=4.8\times10^{-4}\text{ rad}$. If the objects are now a distance *x* apart, their angular separation is $\frac{x}{0.25}$. When magnified, this separation must become $4.8\times10^{-4}\text{ rad}$.

The magnification of the lens is $M=1+\frac{25}{f}=1+\frac{25}{5.0}=6.0$.

So $6.0\times\frac{x}{0.25}=4.8\times10^{-4}\text{ rad}\Rightarrow x=2.0\times10^{-5}\text{ m}$.

23 Using the formula (see page 618 in *Physics for the IB Diploma*),

$$M=-\frac{16}{f_o}\frac{25}{f_e}=-\frac{16}{0.80}\times\frac{25}{4.0}=-125.$$

24 **a** The angular width of the moon is $\theta=\frac{3.5\times10^6}{3.8\times10^8}=0.00921\text{ rad}$.

This is $\theta=0.00921\times\frac{180^\circ}{\pi}=0.527^\circ\approx0.53^\circ$.

b Here $u = 3.8\times10^{8}$ m and so

$$\frac{1}{3.8\times10^{8}}+\frac{1}{v}=\frac{1}{20}\Rightarrow v\approx 20\text{ m}$$

The magnification is then

$$M=-\frac{v}{u}=-\frac{20}{3.8\times10^{8}}=-5.26\times10^{-8}$$

The diameter of the image of the moon is then $5.26\times10^{-8}\times3.5\times10^{6}\approx0.18\text{ m}$.

25 **a** The angular magnification is $M=\frac{f_o}{f_e}=\frac{80}{20}=4$.

b The angle subtended by the building without a telescope is $\theta=\frac{65}{2500}=0.0260\text{ rad}$ and so the angle subtended by the image is $\theta'=M\theta=4\times0.0260=0.104\text{ rad}$.

26 **a** The magnification is $M=\frac{f_o}{f_e}=\frac{67}{3.0}=22.3$.

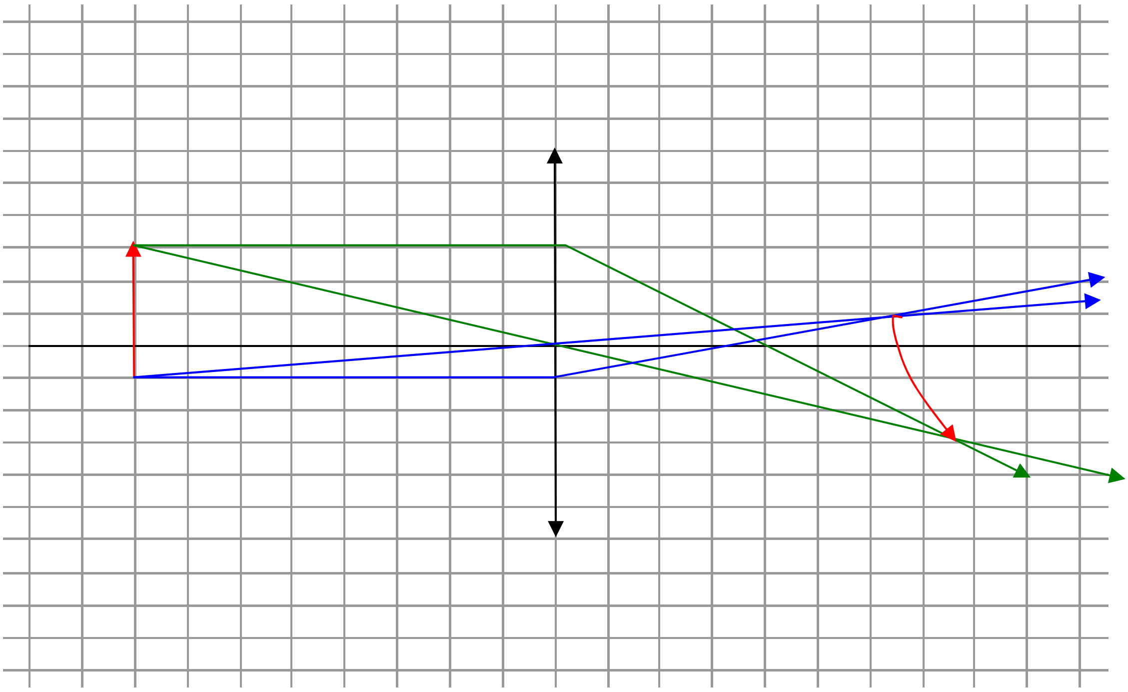
b Its length is $f_o+f_e=70\text{ cm}$.

27 The objective focal length must be 57 cm. If the final image is formed at infinity, it means that the image in the objective is formed at a distance of 3.0 cm from the eyepiece, i.e. $61.5-3.0=58.5\text{ cm}$ from the objective lens.

Hence $\frac{1}{u}+\frac{1}{58.5}=\frac{1}{57.0}\Rightarrow u=2223\text{ cm}\approx22\text{ m}$.

28 **a** There are two main lens aberrations. In spherical aberration, rays that are far from the principal axis have a different focal length from that of rays close to the principal axis. This results in images that are blurred and curved at the edges. In chromatic aberration, rays of different wavelengths have slightly different focal lengths, resulting in images that are blurred and coloured. Spherical aberration is reduced by only allowing rays close to the principal axis to enter the lens, and chromatic aberration is reduced by combining the lens with a second diverging lens.

b **i** The diagram shows (under the simple conditions of this problem) that a different focal length (depending on the distance of the paraxial rays from the principal axis) creates an image that is curved at the edges.



ii The image drawn with one focal length would be straight.

29 We are told that $u = f + x$ and $v = f + y$.

Then $\dfrac{1}{f+x} + \dfrac{1}{f+y} = \dfrac{1}{f} \Rightarrow \dfrac{(f+x)(f+y)}{f+x+f+y} = f$.

Simplifying,

$$f^2 + fx + fy + xy = f(2f + x + y) = 2f^2 + fx + fy$$

$$xy = f^2$$

